1 Curriculum Vitae

Ashot ALEKSIAN

born on 03/09/1996

Postdoctoral Researcher
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Toulouse School of Economics (TSE)

Researcher in applied mathematics for four years, I hold a PhD from the Jean Monnet University in Saint-Étienne and a Master's degree in "Stochastic Modeling and Actuarial Science." I am continuing my research as a postdoctoral researcher at the Toulouse School of Economics. Passionate about stochastic processes, I have strong expertise in various linear and non-linear diffusion processes, their long-term behavior, and the asymptotics of their exit times. I am co-author of two articles published in "ESAIM: PS" and the "Electronic Journal of Probability," as well as an article under revision in "Probability Theory and Related Fields."

Education

- **2020 - 2023**
  PhD in Applied Mathematics, University of Jean Monnet, Saint-Étienne, France:
  Exit-problem for Self-interacting and Self-stabilizing Diffusion Processes
  defended on 20/11/2023 before the jury composed of:

Champagnat, Nicolas	DR / Inria Nancy-Grand Est	Chair
Essouabri, Driss	PR / UJM Saint-Étienne, ICJ	Examiner
Herrmann, Samuel	PR / Université de Bourgogne, IMB	Examiner
Miclo, Laurent	DR / CNRS	Examinateur
Koralov, Leonid	Full professor / University of Maryland	Reviewer/Examiner
Lelievre, Tony	PR / École des Ponts, CERMICS	Reviewer/Examiner
Tugaut, Julian	MCF / UJM Saint-Étienne, ICJ	PhD Supervisor
Kurtzmann, Aline	MCF / Université de Loraine, IECL	PhD Co-supervisor

- **2017 - 2019**
  Master. Specialization in "Stochastic Modeling and Actuarial Science"
  Moscow, Russia
  Higher School of Economics (HSE University)
  Master's thesis: "Recent applications of Optimal Mass Transportation"
  Supervisor: Jean-François Jabir

- **2013 - 2017**
  Bachelor. Specialization in "Applied Informatics"
  Moscow, Russia
  Higher School of Economics (HSE University)

Professional Experience

- **November 2023 - Present**
  Toulouse School of Economics
  Toulouse, France
  Postdoctoral Researcher
  Topic: Studying the convergence and speed of convergence of a new algorithm (for finding the global minima of a function) based on a McKean-Vlasov type diffusion process.
  Supervisors: Stéphane Villeneuve, Laurent Miclo.

- **2020 – 2023**
  Jean Monnet University
  Saint-Étienne, France
  PhD Candidate
  Research Focus: Solving the exit time and exit location problems for self-interacting diffusions and McKean-Vlasov-type diffusions. The Kramers-type law was established: the exit time from a domain of attraction has an exponential form. To achieve this in various contexts, numerous techniques were employed, including large deviations principles and a new coupling method.
  Supervisors: Aline Kurtzmann, Julian Tugaut.

- **2018 - 2020**
  Glowbyte Consulting
  Moscow, Russia
  Analyst
  Tasks: Development and implementation of databases and other information systems for risk management.
  Projects: Participated in several consulting projects for a leading bank in Russia. These projects included the development and integration of PD, LGD, and EAD models for assessing risks related to small and medium-sized enterprise debtors.

Publications et prépublications

1. Ashot Aleksian, Pierre Del Moral, Aline Kurtzmann, et Julian Tugaut. On the exit-problem for self-interacting diffusions. [ESAIM: PS 28: 46-61] 2024. `https://doi.org/10.1051/ps/2023020`
2. Ashot Aleksian, Aline Kurtzmann, et Julian Tugaut. Exit-problem for a class of non-Markov processes with path dependency. [en révision pour Probability Theory and Related Fields] 2023. `https://arxiv.org/abs/2306.08706`
3. Ashot Aleksian, et Julian Tugaut. Measure-dependent non-linear diffusions with superlinear drifts: existence, large deviations principle and asymptotic behavior of the first exit-times. [Electronic Journal of Probability 29: 1-31] 2024. `https://doi.org/10.1214/24-EJP1229`

Publications and Preprints

1. Ashot Aleksian, Pierre Del Moral, Aline Kurtzmann, and Julian Tugaut. On the exit-problem for self-interacting diffusions. [ESAIM: PS 28: 46-61] 2024. `https://doi.org/10.1051/ps/2023020`
2. Ashot Aleksian, Aline Kurtzmann, and Julian Tugaut. Exit-problem for a class of non-Markov processes with path dependency. [under revision for Probability Theory and Related Fields] 2023. `https://arxiv.org/abs/2306.08706`
3. Ashot Aleksian, and Julian Tugaut. Measure-dependent non-linear diffusions with superlinear drifts: existence, large deviations principle and asymptotic behavior of the first exit-times. [Electronic Journal of Probability 29: 1-31] 2024. `https://doi.org/10.1214/24-EJP1229`

Research Projects

- **2023 – Present**
  Convergence of swarm gradient dynamics
  Part of the USAF FA8655-22-1-7016 grant
  Objective: Study the convergence and the speed of convergence of a new algorithm (for finding the global minima of a function) based on a McKean-Vlasov type diffusion process.
  Principal Investigators: Laurent Miclo (CNRS, Toulouse School of Economics), Stéphane Villeneuve (Toulouse School of Economics).

- **2021 - 2023**
  PHC SAKURA 2021 (code: 47005SC)
  Before, During and After the Blow-Up in the Analysis of Chemotaxis Models: Macroscopic and Microscopic Viewpoints
  Objective: Combine PDE and probabilistic techniques to analyze the phenomenon of "blow-up" in chemotaxis models. Blow-up is a phenomenon in PDE solutions where certain quantities become infinite in finite time. Chemotaxis is a collective movement of a population of cells or bacteria triggered by a chemical stimulus in their environment, which can be either attractive or repulsive.
  Participants: Ashot Aleksian, Milica Tomašević (École Polytechnique), Julian Tugaut (Université Jean Monnet, French coordinator), Kentaro Fujie (Tohoku University, Japanese coordinator), Hironari Miyoshi (Saitama University), Hiroshi Wakui (Tokyo University of Science).

- **2019 - 2024**
  ANR METANOLIN (ANR-19-CE40-0009)
  METAstability for NOnLINear processes (JCJC)
  Objective: Address metastability problems associated with non-linear stochastic processes (including dependencies on the law or the history of the process itself). Metastability issues arise in complex optimization problems when using stochastic gradient descent methods. The project aims to leverage non-linearity to develop algorithms faster than standard methods.
  Participants: Ashot Aleksian, Paul-Éric Chaudru de Raynal (Université de Nantes), Aline Kurtzmann (Université de Lorraine), Pierre Monmarché (Sorbonne Université), Milica Tomašević (École Polytechnique), Julian Tugaut (Université Jean Monnet, principal investigator).

Talks

1. Presentations at Conferences

- **November 2024**
  "LSA Winter Meeting" Workshop
  Online; Moscow, Russia

- **June 2024**
  Journées de Probabilités 2024
  Bordeaux, France

- **June 2024**
  TSE Doctoral Workshop
  Toulouse, France

- **June 2023**
  Workshop "Stochastic Processes, Metastability and Applications"
  Nancy, France

- **October 2022**
  First Franco-Japanese Workshop on Chemotaxis Models (Macroscopic and Microscopic Viewpoints)
  Sendai, Japan

- **May 2022**
  Workshop “Metastability, Mean-Field Particle Systems and Nonlinear Processes”
  Saint-Étienne, France

2. Seminar Presentations

- **April 2024**
  SPOC Team Seminar, IMB
  Dijon, France
  Title: “Exit-time for McKean-Vlasov Processes: Beyond the Convex Case”

- **March 2024**
  Seminar of the Probability, Analysis, and Statistics Team
  Clermont-Ferrand, France
  Title: “Exit-time for McKean-Vlasov Processes: Beyond the Convex Case”

- **February 2024**
  Probability Seminar, IRMAR
  Rennes, France
  Title: “Metastability for Non-linear Processes: New Results and Open Questions”

- **February 2024**
  ProbaStat Team Seminar
  Poitiers, France
  Title: “Metastability for Non-linear Processes”

- **October 2023**
  Seminar “Stochastic Analysis and Applications”
  Moscow, Russia
  Online presentation. Title: “Exit-Time Problem for Self-Interacting and Self-Stabilizing Diffusion Processes”

- **April 2022**
  Working Group
  Nancy, France
  Title: “Exit-Time Problem for Self-Interacting and Self-Stabilizing Diffusion Processes”

3. Conference Participation

- **September 2023**
  A Random Walk in the Land of Stochastic Analysis and Numerical Probability
  Marseille, France
  Poster

- **August 2023**
  Workshop “Celebrating the Mathematics of Michel Benaïm”
  Lausanne, Switzerland

- **June 2023**
  Summer School on Mean Field Models
  Rennes, France

- **September 2022**
  Interacting Particle Systems and Applications
  Trento, Italy

Activités Pédagogiques

- **2024**
  TD d'Optimisation
  Toulouse, France
  15h de TD pour L3 Économie.

- **2024**
  TD de Statistique Inférentielle
  Toulouse, France
  15h de TD pour L2 Économie.

- **2022**
  Stage hippocampe
  Saint-Étienne, France

- **2019**
  Calculus III
  Moscou, Russie
  Assistant du professeur. Donner des notes pour les devoirs et examens.

Other Information

Languages:

- English: Fluent
  Certificate: Cambridge English Exam, Level: C1 Advanced

- French: Fluent
  Certificate: CILEC Saint-Étienne, Level: B2

- Russian: Native

Nationality: Russian.

Software Skills: Python, C/C++, R, MATLAB, LaTeX, SQL.

Qualification: MCF-2024-26-24226400134. Maître de conférences (MCF) qualification in section "26 - Applied Mathematics and Applications of Mathematics."

2 Summary of my previous works

Keywords: *McKean-Vlasov processes, Self-interacting diffusions, Exit-time problem, Freidlin-Wentzell theory, Large deviations theory, Stochastic differential equations, Stochastic processes.*

My previous work consists of three articles written in collaboration with Pierre Del Moral, Aline Kurtzmann, or Julian Tugaut:

[AdMKT24] Ashot Aleksian, Pierre del Moral, Aline Kurtzmann, and Julian Tugaut. Self-interacting diffusions: Long-time behaviour and exit-problem in the uniformly convex case. *ESAIM: PS*, 28:46–61, 2024.

[AKT23] Ashot Aleksian, Aline Kurtzmann, and Julian Tugaut. Exit-problem for a class of non-Markov processes with path dependency, 2023. under review for PTRF.

[AT24] Ashot Aleksian and Julian Tugaut. Measure-dependent non-linear diffusions with superlinear drifts: asymptotic behaviour of the first exit-times. *Electronic Journal of Probability*, 29:1 – 31, 2024.

In these articles, the exit-time problem for two types of nonlinear diffusion processes is studied. The first article [AdMKT24] considers the self-interacting diffusion (denoted SID), defined by the following stochastic differential equation, which includes the interaction of the process with its own past:

$$\mathrm{d}X_t^\sigma = \sigma\,\mathrm{d}W_t - \left(\nabla V(X_t^\sigma) + \frac{1}{t}\int_0^t \nabla F(X_t^\sigma - X_s^\sigma)\,\mathrm{d}s\right)\mathrm{d}t\,,$$

where W_t is the Brownian motion, and $\sigma > 0$ is a parameter controlling the noise intensity.

In this first article, we study the exit-time problem for this process, i.e., the first time the diffusion exits a fixed domain G in the small noise regime. Furthermore, we assume there exists a unique attraction point within the domain G, which we denote by a. Consequently, the exit-time problem corresponds to studying the following stopping time:

$$\tau^\sigma := \inf\{t \geq 0 \ : \ X_t^\sigma \notin G\}.$$

In [AdMKT24], we assume that the functions V and F are convex, which allows us to use classical coupling techniques to establish Kramers' law and the exit localization result. In particular, under the article's hypotheses, we prove the following theorem:

Théorème 1. *Define the exit cost as:*

$$H := \inf_{x\in\partial G}\left(V(x) + F(x-a) - V(a)\right)\,.$$

For any $\delta > 0$, we have Kramers' law, that is:

$$\lim_{\sigma\to 0}\mathbb{P}\left(\exp\left\{\frac{2}{\sigma^2}(H-\delta)\right\} \leq \tau^\sigma \leq \exp\left\{\frac{2}{\sigma^2}(H+\delta)\right\}\right) = 1\,.$$

Moreover, if $\mathcal{N} \subseteq \partial G$ and satisfies $\inf_{z\in\mathcal{N}}\left(V(z) + F(z-a) - V(a)\right) > H$, then

$$\lim_{\sigma\to 0}\mathbb{P}\left(X_{\tau^\sigma} \in \mathcal{N}\right) = 0\,.$$

The second part of the theorem corresponds to the so-called exit localization.

To prove this theorem, we first recall the existing results on the convergence of the occupation measure of the process $\mu_t^\sigma = \frac{1}{t}\int_0^t \delta_{X_s^\sigma}\,\mathrm{d}s$ toward a Gibbs measure. We use this result to demonstrate the stabilization of the occupation measure around the attraction point δ_a in finite time. This stabilization and the convexity of the potentials V and F are used to derive a coupling result between our system and the associated Itô diffusion:

$$\mathrm{d}Y_t^\sigma = \sigma\,\mathrm{d}W_t - \Big(\nabla V(Y_t^\sigma) + \nabla F(Y_t^\sigma - a)\Big)\,\mathrm{d}t \tag{1}$$

until the exit time. This coupling allows us to show that the exit time of the self-interacting diffusion corresponds to that of the associated Itô process. The same holds for the exit localization result.

In the article [AKT23], we significantly improved this result by considering more general confinement and interaction conditions. However, we assumed that V and F were sufficiently regular with locally Lipschitz gradients. This greatly complicated the problem because, in this case, we could not control the occupation measure or even couple the process with the associated Itô diffusion in the same way. Thus, to prove the exit-time result, we had to adapt Freidlin-Wentzell theory to this system.

Our analysis began with a "Markovization" of the self-interacting diffusion. Although the SID is not a Markov process, to determine the future trajectory of our process, only the entire past trajectory is necessary. Note that the past trajectory can be described by the following triplet (t_0, μ_0, x_0), where t_0 is its duration, μ_0 is the occupation measure, and x_0 is the last point, while the future trajectory is defined by the SDE:

$$\begin{cases} \mathrm{d}X_t^\sigma &= -\nabla V(X_t^\sigma)\,\mathrm{d}t - \nabla F * \mu_t^\sigma(X_t^\sigma)\,\mathrm{d}t + \sigma\,\mathrm{d}W_t\,, \\ \mu_t^\sigma &= \frac{t_0}{t_0+t}\mu_0 + \frac{1}{t_0+t}\int_0^t \delta_{X_s^\sigma}\,\mathrm{d}s\,, \\ X_0^\sigma &= x_0 \;\text{ a.s.} \end{cases} \tag{2}$$

We tackled this problem by first proving the large deviations principle (denoted LDP) for the generalized system above. We used the LDP to control the occupation measure up to the exit time and restored the logic of Freidlin-Wentzell theory to prove the exit-time result. Specifically, we considered cases where the diffusion approaches the attraction point a and when it deviates significantly from it. We showed that the time spent by the process around the point a is significantly greater than the time spent far from it. This provided us with control of the occupation measure.

To obtain the exit-time result, following the approach of Freidlin and Wentzell, we again studied the process when it approaches a and when it moves away from it. Controlling the empirical measure of the process implies that each deviation can be considered as an independent exit attempt of processes of the type (2) with μ_0 close to δ_a. Similarly to the Itô diffusion, the process exits the domain G along the least improbable path, i.e., the one that minimizes the quasipotential. As we highlighted above, $\mu_t^\sigma \xrightarrow[\sigma\to 0]{} \delta_a$ for all t at least until the exit time, and with $\sigma \to 0$, the quasipotential becomes closer to that of the Itô process (1), which helped us establish Kramers' law of the form:

$$\lim_{\sigma\to 0} \mathbb{P}\left(\exp\left\{\frac{2}{\sigma^2}(H-\delta)\right\} \le \tau^\sigma \le \exp\left\{\frac{2}{\sigma^2}(H+\delta)\right\}\right) = 1,$$

where $H := \inf_{x\in\partial G}(V(x) + F(x-a) - V(a))$. Furthermore, an exit localization result similar to that of Theorem 1 was established.

The article [AT24] studies the exit problem for the self-stabilizing diffusion (denoted SSD) with general confinement and interaction potentials. We considered the following process:

$$\begin{cases} \mathrm{d}X_t^\sigma &= -\nabla V(X_t^\sigma)\,\mathrm{d}t - \nabla F * \nu_t^\sigma(X_t^\sigma)\,\mathrm{d}t + \sigma\,\mathrm{d}W_t\,, \\ \nu_t^\sigma &= \mathcal{L}(X_t^\sigma), \\ X_0^\sigma &= x_0 \in \mathbb{R}^d \;\text{ a.s.,} \end{cases} \tag{3}$$

where $\mathcal{L}(X_t^\sigma)$ represents the law of the random variable X_t^σ.

The formulation of this process is very similar to that of the self-interacting diffusion. Furthermore, after controlling the law of the SSD, we could have used the same technique as in the article [AKT23] to establish the exit-time result. Instead, we developed techniques to generalize the coupling method and apply it to our non-convex case.

Similarly to [AKT23], we first show the finite-time convergence of the process X^σ to the attraction point a. After this convergence, we introduce a synchronous coupling of X^σ with an Itô diffusion (1). The issue here is that we can no longer use the convexity of V or F, since we consider them to be more general.

The idea is as follows: since the processes X^σ and Y^σ are coupled by the same Brownian motion, whenever X^σ and Y^σ are close to the stable attraction point a (in the small neighborhood where $V+F(\cdot-a)$ is assumed to be convex), the distance between them almost surely decreases. At the same time, whenever the two processes deviate from a but remain inside the domain G, their largest deviation can be controlled based on the time spent in the confined ring between ∂G and a small neighborhood around a. We then prove that the processes X^σ and Y^σ spend enough time near a, relative to the total time spent away from it, to the point that the attractive effect outweighs the repulsive effect. We use this to prove that for all $\kappa > 0$:

$$\lim_{\sigma\to 0} \mathbb{P}(\sup_t |X_t^\sigma - Y_t^\sigma| > \kappa) = 0,$$

where the supremum is taken over t in an interval sufficiently large to prove the exit-time result.

This coupling allows us to control the law $\mathcal{L}(X_t^\sigma)$ at least up to the exit time. Moreover, it enables us to prove Kramers' law as well as the exit localization result for the SSD (3).